



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

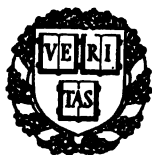
Wentworth. Answers to Algebra
1889

EducT
128
89.876



3 2044 097 011 860

EducT 128 . 89 . 876



HARVARD
COLLEGE
LIBRARY

THE GIFT OF

*Miss Ellen Lang Wentworth
of Exeter, New Hampshire*

° ANSWERS

TO THE PROBLEMS IN

WENTWORTH'S COLLEGE ALGEBRA.



BOSTON, U.S.A.:

PUBLISHED BY GINN & COMPANY.

1889.

Eding T 128.89.876

HARVARD COLLEGE LIBRARY
GIFT OF
MISS ELLEN L. WENTWORTH

MAY 8 1939

ANSWERS

TO

EXERCISES IN THE COLLEGE ALGEBRA.

Ex. 1. Page 10.

1. $5a^2 + 4a + 7b$.
3. $-12a^3 + 7a^2b + 11ab^2 - 11b^3$.
2. $15x^2 - 7xy + 11y^2$.
4. $x^5 + 2x^4 + 12x^3 + 9x^2 - 10x - 12$.
5. $8m^4 + 10m^3n - 7m^2n^2 - 7mn^3 - 9n^4$.
6. $2x^6 + 11x^5y - 11x^4y^2 + x^3y^3 - 3xy^5 - 7y^6$.

Ex. 2. Page 13.

1. $2a - 4b + 5c$.
5. $5x^3 + 10x^2 - 3x - 18$.
2. $5x^3 - 9x^2 + 13x - 10$.
6. x .
3. $a^3 + 8a^2b - 16ab^2 + 12b^3$.
7. $5b - 5c$.
4. $3a^3 + \frac{1}{8}ab - \frac{3}{16}b^2 + \frac{1}{8}a$.
8. $2a - 4b$.
9. $-10a + 15b$.
10. $2xy + xz - 2yz - y^2 + xz^2 - x^2z$.
11. $-2x^4 - 15ax^3 + 8a^2x^2 + 6a^3x - 5a^4$.

Ex. 3. Page 18.

1. $12x^2 - 7xy - 10y^2$.
2. $8x^3 + 6x^2 - 20x - 15$.
3. $4x^4 + 14x^3 - 2x^2 - 25x + 12$.

4. $x^3 + 4x^2 + 16.$
5. $x^4 - 3x^2y - 9x^2y^2 + 23xy^3 - 12y^4.$
6. $27x^3 - y^3 + 18xy + 8.$
7. $33a^6 - a^5b - 44a^2b^4 - 80ab^5 - 16b^6.$
8. $8a^3b + 8ab^3.$
9. $x^2 + 4y^2 + 9z^2 - 4xy + 6xz - 12yz.$
10. $x^5 + 4x^3 - 4x^2 - 18x^3 + 12x^2 + 8x + 1.$
11. $1170d^{3p+1} - 1620d^3 + 1800d^{5p+2}.$
12. $600x - 1050x^{m-5n+4} + 625x^{2m}.$
13. $2a^{p+3} - 7a^{p+2} + 12a^{p+1} - 19a^p + 20a^{p-1} - 11a^{p-2} + 5a^{p-3}.$
14. $a^{2n+3} - 2a^{2n+3} - 2a^{2n+3} + 2a^{2n+1} + a^{n+3} + 2a^{n+2} - a^{n+1} - 2a^n + a^{n-1}.$
15. $a^{2p+2} - 4a^{2p} + 12a^{2p-1} - 9a^{2p-2}.$

Ex. 4. Page 24.

1. $\frac{5}{abc^4}.$
2. $\frac{1}{2}a + 4a^2x + 7x + \frac{3}{2}a^2.$
3. $7x^2 + 16ax + 4a^2.$
4. $x - 2a.$
5. $9x^2 + 6xy + 4y^2.$
6. $x^2 + b^2 - ax.$
7. $a + 2b - c.$
8. $2x^2 + 3xy + y^2 - 2.$
9. $a^n - a^m.$
10. $125x^3 + 75x^2y + 45xy^2 + 27y^3.$
11. $x^{2n} - x^ny^n + y^{2n}.$
12. $\frac{9a^2}{25} + \frac{3ab}{20} + \frac{b^2}{16}.$
13. $a^2 + 3ab + 3b^2 + 3ac + 9c^2.$
14. $a^m + 2a^{m+1} - 4a^{m+2} + 5a^{m+3}.$
15. $4h^4 - 2h^3 - 3h^2.$
16. $2x^{m-1} + 3x^{m-2} - 4x^{m-3}.$

Ex. 5. Pages 32, 33.

1. $x(3x+2)(3x^2+1)$.
2. $a(2a-3b)(a^2-7)$.
3. $(x+3y)(5x^2-4y^2)$.
4. $(ax-by)(ax+by)(x-c)$.
5. $(x+7)(x+1)$.
6. $(x-12)(x-5)$.
7. $(x+9)(x-2)$.
8. $(x-6)(x+4)$.
9. $(3x+5)(3x+5)$.
10. $(4x-7)(4x-7)$.
11. $(x+9)(x-8)$.
12. $(x-22)(x+8)$.
13. $x^2(9x-14y)(9x+14y)$.
14. $(3a+x)(3a-x)(9a^2-3ax+x^2)(9a^2+3ax+x^2)$.
15. $x(4x^2+y^2)(16x^4-4x^2y^2+y^4)$.
16. $x^2(x^2-2y^2)$.
17. $(a^2+ab+2b^2)(a^2-ab+2b^2)$.
18. $(x-y)(3x-5y)$.
19. $(x-1)(x-7)(3x^2+7)$.
20. $(x^2-b^2+c^2)(x^2-b^2+c^2)$.
21. $(3x-2)(5x+1)$.
22. $(x-3)(11x-21)$.
23. $(3x+5)(7x-3)$.
24. $(5x-3)(14x+3)$.
25. $(x^2+a^2-ab+b^2)(x^2-a^2-ab-b^2)$.
26. $(x^2-5)(5x^2+4x+25)$.
27. $(2x^2-x+1)(x^2-2x-2)$.
28. $(2x^2-ax+a^2)(x+a)(3x-2a)$.
29. $2(x+y)(x-y)(6x^2+5x^2y+2y^2)$.

Ex. 6. Page 40.

1. $3x-2$.
2. $x-a$.
3. x^2-4x+4 .
4. $3x^2-x+4$.
5. $4x-1$.
6. $x-2$.
7. $x+3$.
8. $2a^2-3ax$.
9. a^3-2a^2+a .
10. $3x-y$.
11. $6x^3+2x^2-4x$.

12. $x^4 + 4x^3 - 13x^2 - 64x - 48$.
 13. $36x^4 - 97x^3 + 36$.
 14. $6x^5 + 13x^4 - 9x^3 - 6x^2 - 6x - 40$.
 15. $60a^3x^5 + 190a^4x^5 + 116a^5x^4 + 64a^6x^3 + 48a^7x^2 - 30a^8x$.
 16. $4x^5 + 8x^4 - 13x^3 - 26x^2 + 9x + 18$.

Ex. 7. Pages 46, 47.

1. $\frac{6a^2}{5x^2}$ 4. $\frac{x^2 - ax + b^2}{x^2 + ax - b^2}$ 7. $\frac{a^2 - x^2}{2a^2x^2}$.
 2. $\frac{2x - 3}{7x - 3}$ 5. $\frac{3x^2 - 2}{2x^3 + 8x^2 + 12x}$ 8. $\frac{3x^3 - 2ax - 6a^2}{(x - a)^3}$.
 3. $\frac{3c^2 - a^2}{2a^3 + c^2}$ 6. 0 9. $-\frac{2a}{(a - b)(b - c)}$ 10. $\frac{1}{abc}$.
 11. 4. 14. a . 17. $-\frac{4x^2(x + y)^2}{y^2(2x + y)^2}$.
 12. $\frac{9bc}{4x}$ 15. $\frac{2x - 1}{3x + 2}$ 18. $\frac{1 + x^4}{x + x^3}$.
 13. $\frac{x^2 - xy}{y}$ 16. 1. 19. $a^4 - a^2b^2 + b^4$.
 20. $\frac{8a(4a - 3x)(2a - 9x)}{27(4a + 3x)(2a + 9x)}$

Ex. 8. Pages 52, 53.

1. 5. 2. True for all values of x .
 3. 8. 7. -2. 11. 8.
 4. 3. 8. 5. 12. 7.
 5. 4. 9. $\frac{3}{4}$. 13. 3.
 6. $-1\frac{1}{11}$. 10. 4. 14. 2.

15. $3\frac{5}{11}$. 16. 2. 17. $\frac{11}{13}$. 18. $3\frac{2}{3}$. 19. $-\frac{4}{11}$.
 20. $\frac{5a^2 + 4a}{4a + 2}$. 21. $\frac{a^2bc^2 + ab + c^2}{a^2 - bc}$.
 22. $2a - 3b$. 23. $4a$. 24. $4a$.

Ex. 9. Pages 55, 56.

1. 12, 9. 3. 16. 5. 16 hours.
 2. 12. 4. $\frac{7}{12}$. 6. $\frac{ac}{b-a}$ hours.
 7. 25 miles per hour. 11. $2\frac{2}{3}$ miles per hour.
 8. $23\frac{1}{2}$ days. 12. $\frac{ma(c-b)}{(m+1)bc}$ miles per hour.
 9. $\frac{ab}{a-b}$ days. 13. 15 miles; 2 miles per hour.
 10. $\frac{4m^2n + 2mn^2}{4m^2 + 4mn - n^2}$ days. 14. $\frac{nab}{b-a}$ feet.
 15. $\frac{ml - bl}{a-b}$ gals., $\frac{al - ml}{a-b}$ gals.

Ex. 10. Pages 61-83.

The upper number is the value of x , the lower that of y .

1. 6. 6. 34. 11. 3.
 2. 46. 4.
 2. 12. 7. 9. 12. $\frac{1}{8}$.
 4. 8. $\frac{1}{4}$.
 3. 13. 8. 12. 13. $\frac{4123}{288}$.
 3. 5. $\frac{442}{36}$.
 4. -3. 9. 10. 14. -3.
 -2. 5. 122.
 5. 8. 10. 9. 15. $a+b$.
 2. 7. $b-a$.

- | | |
|----------------------------|---|
| 7. $1 + 2x + 3x^2$. | 11. $5x + 3y - 2$. |
| 8. $3 - 4x - 14x^2$. | 12. $2x^2 - 3y^2 + 4$. |
| 9. $2 - 3x - x^2 + 5x^2$. | 13. $\frac{x^2}{2} - \frac{y^2}{3} + \frac{1}{4}$. |
| 10. $6x^2 - 10a^2 - a^4$. | 14. 18.0554. |
| 15. 31.9374. | 16. 1.9147. |
| | 17. 1.1455. |

Ex. 14. Page 76.

- | | |
|--------------------------------|-----------------------------|
| 1. $3 - 4x$. | 4. $1 - 2x + 3x^2$. |
| 2. $x^2 - x - 1$. | 5. $3 - 4x - 5x^2$. |
| 3. $a - \frac{b}{3}$. | 6. $2x + 1 - \frac{5}{x}$. |
| 7. $2x - 3a + \frac{a^2}{x}$. | 8. 8.026. |
| 9. 11.786. | 10. 1.480. |
| | 11. 27.550. |

Ex. 15. Page 80.

5. $a^{-\frac{1}{2}}$; $b^{\frac{1}{2}}$; $c^{\frac{1}{2}}$.
6. $a^{\frac{1}{2}}$; $\frac{bc^{\frac{1}{2}}}{x^{\frac{1}{2}}}$; $a^{\frac{1}{2}}x^{\frac{1}{2}}$.
7. $64x^{\frac{1}{2}}(3a)^{\frac{1}{2}}$; $\frac{27a^3x^{\frac{1}{2}}}{8}$; $\frac{64}{27a^3x^{\frac{1}{2}}}$; $\frac{3^{\frac{1}{2}}x^{\frac{1}{2}}}{27a^4}$.
8. $x^2 + 1$.
9. $x^{4p} + x^{2p}y^{2p} + y^{4p}$.
10. $16a + 8a^{\frac{1}{2}}b^{-\frac{1}{2}} + 10a^{\frac{1}{2}}b^{-\frac{3}{2}} + 18a^{\frac{1}{2}}b^{-\frac{5}{2}} - 8a^{\frac{1}{2}}b^{-\frac{7}{2}} - 4a^{\frac{1}{2}}b^{-\frac{9}{2}} - 5a^{\frac{1}{2}}b^{-\frac{11}{2}} - 9b^{-1}$.
11. $x^{4n} - x^{3n}y^n + x^{2n}y^{2n} - x^n y^{3n} + y^{4n}$.
12. $x^{\frac{1}{2}} - y^{-1}$.
13. $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}c^{-\frac{1}{3}} + b^{\frac{2}{3}} - b^{\frac{1}{3}}c^{-\frac{1}{3}} + c^{-\frac{1}{3}}$.

Ex. 16. Pages 83, 84.

5. $\sqrt{\frac{xz}{y}}$; $\frac{a^2b^3\sqrt[3]{a^3}}{c}$; $2a^2b^4x\sqrt[3]{b^3}$.
8. $6\sqrt{7} > 5\sqrt{10} > 9\sqrt{3}$.
9. $3\sqrt[3]{5} < 4\sqrt[3]{4} < 5\sqrt[3]{3}$.
10. $24\sqrt{3}$; 2.
11. $\frac{2\sqrt{3}}{9}$; $\frac{4\sqrt{15}}{3}$.
12. $4\frac{2}{3}$.
13. $3\sqrt{2} > \frac{5}{2}\sqrt[4]{4} > 2\sqrt[3]{3}$.
14. $3\sqrt{19} > 5\sqrt[3]{2} > 3\sqrt[3]{3}$.
15. $\sqrt[20]{a^{18}x^9y^{19}}$; $\frac{3\sqrt[6]{2a^6b}}{a^2}$.
16. $\frac{1}{80}$; $a^3b^7\sqrt[7]{b^3}$.

Ex. 17. Page 86.

1. $29\sqrt{3}$; $33\sqrt[3]{2}$.
2. $\frac{2}{3}\sqrt{3}$; $4\frac{2}{3}\sqrt{15}$.
3. $\left(\frac{a^2}{b^2} - \frac{ac}{bd} - \frac{ad}{bm}\right)\sqrt{bc}$; $\frac{2}{3}\sqrt{10}$.
4. 0.
5. $2\sqrt[3]{2}$; $3\sqrt[3]{3}$; $4\sqrt[4]{2}$; $2\sqrt[3]{2}$.
6. a^{-4} ; $x^{-\frac{2}{3}}$; $p^{\frac{1}{2}}$; $a^{\frac{1}{15}}$.
7. $x^{2m} + 3x^my^n + y^{2n}$.
8. $1 + 2x^{-\frac{1}{2}} - 3x^{-\frac{3}{2}} + 4x^{-1}$.

9. $x^{\frac{1}{2}}$.
 10. $\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{c}$.
 11. $\frac{2b^2}{25a^{\frac{1}{2}}c^2}$.
 12. $\frac{5 \times 5^{\frac{1}{2}}a^{\frac{1}{2}}}{2b^{\frac{1}{2}}}$.
 13. x^{pq} .
 14. $x^{pq+p} - y^{pq-q}$.
 15. 0.614.....; 3.461.....; 1.071.....; 59.394.....
 16. 1.154.....; 1.216.....; 3.576.....; 1.449.....

Ex. 18. Pages 89, 90.

1. $\pm \sqrt{3}$.
 2. $\pm \frac{1}{2}$.
 3. $\pm \frac{1}{2}$.
 4. $\pm \sqrt{11}$.
 5. ± 5 .
 6. ± 3 .
 7. ± 5 .
 8. ± 2 .
 9. $\pm \sqrt{-\frac{a}{1+b^2}}$.
 10. $\pm \sqrt{\frac{c-b}{a}}$.
 11. $\pm \sqrt{\frac{b}{a-1}}$.
 12. $\pm b$.
 13. $\pm a$.
 14. $\pm 3a$.
 15. $\pm \frac{(a+b)\sqrt{3}}{2}$.

Ex. 19. Pages 95-97.

1. 5; -3.
 2. 8; 6.
 3. 4; -3.
 4. 7; -4.
 5. 7; 6.
 6. 9; 12.
 7. $1\frac{1}{2}$; -2.
 8. $1\frac{1}{4}$; -3.
 9. 4; $2\frac{1}{3}$.
 10. $\frac{3}{4}$; -5.
 11. $1\frac{1}{2}$; $-1\frac{1}{3}$.
 12. $\frac{2}{3}$; $1\frac{1}{3}$.
 13. $\frac{1}{3}$; $\frac{5}{3}$.
 14. $-1\frac{2}{3}$; $-1\frac{1}{3}$.
 15. 14; -1.
 16. 6; -10.
 17. 5; $-2\frac{1}{2}$.
 18. 3; -9.
 19. 4; $-2\frac{1}{2}$.
 20. 2; 9.
 21. 3; -8.
 22. 3; $-\frac{4}{3}$.
 23. 7; -2.
 24. 6; $-\frac{1}{2}$.

- | | | |
|---------------------------|--------------------------|-------------------------------------|
| 25. $3; \frac{3}{5}$. | 28. $1; -1\frac{1}{4}$. | 31. $\frac{3}{11}; -\frac{3}{11}$. |
| 26. $13; -4\frac{1}{2}$. | 29. $5; 1\frac{1}{2}$. | 32. $8; -1$. |
| 27. $4; -5$. | 30. $5; -1\frac{1}{4}$. | 33. $1; -2\frac{8}{13}$. |

Ex. 20. Pages 98-100.

- | | |
|---|--|
| 1. $3a; -a$. | 16. $a; -b$. |
| 2. $a; 7a$. | 17. $a; 3a+b$. |
| 3. $\frac{a+b}{2}; \frac{a-b}{2}$. | 18. $2b; \frac{ab}{2a-b}$. |
| 4. $6a; -\frac{4a}{3}$. | 19. $\frac{a}{a-b}; \frac{b}{a-b}$. |
| 5. $\frac{a \pm \sqrt{a^2 + 4b}}{2}$. | 20. $4a-3b; 3b-a$. |
| 6. $\frac{a(a+b)}{a-b}; \frac{a(a-b)}{a+b}$. | 21. $\frac{3(a+b)}{2}; -\frac{a+b}{2}$. |
| 7. $\frac{3}{2a}; -\frac{1}{2a}$. | 22. $-4a; -\frac{3a}{2}$. |
| 8. $a; b$. | 23. $a-b; -(a+2b)$. |
| 9. $\frac{m}{n}; \frac{n}{m}$. | 24. $a+2b; a-2b$. |
| 10. $\frac{a}{2}; \frac{3a}{4}$. | 25. $\frac{a+3b}{2}; -\frac{3a+b}{2}$. |
| 11. $\frac{a}{2}; \frac{b}{2}$. | 26. $\frac{2a+3b}{2a-3b}; \frac{2a-3b}{2a+3b}$. |
| 12. $2m; \frac{m}{2}$. | 27. $\frac{a+b}{a-b}; \frac{a-b}{a+b}$. |
| 13. $\frac{-5a \pm \sqrt{25a^2 - 9a}}{2}$. | 28. $\frac{2a+b}{2a-b}; \frac{2a-b}{2a+b}$. |
| 14. $ab \pm a\sqrt{b^2 - b}$. | 29. $a-2b; \frac{4b^2}{a+2b}$. |
| 15. $a; a+b$. | 30. $\frac{a-b+c}{a-b}; -2$. |

31. $c; 2a - 3b - c.$
 32. $a + b; a^2 - b^2.$
 33. $2m(2m - n);$
 $n(2m + n).$
 34. $m + n; \frac{1}{m} + \frac{1}{n}.$
 35. $\frac{b - a}{2a - 3b}; \frac{b - a}{2a + 3b}.$
 36. $a + 4b; 2(a - b).$
 37. $a - 8b; 2(a + 3b).$
 38. $\frac{a - 6b}{2}; -(5a + b).$
 39. $-(2a + 3b);$
 $\frac{4a^2 + 9b^2}{2a + 3b}.$
 40. $\frac{a - b}{a - 2b}; -\frac{a + b}{a + 2b}.$
 41. $1; \frac{2a + 3b}{2a - 3b}.$
 42. $2b - a; \frac{(a + 2b)^2}{a - 2b}.$
 43. $\frac{a - b}{2a - 3b}; \frac{a + b}{2a + 3b}.$
 44. $a - b; 9a - b + 2c.$

Ex. 21. Page 103.

Integral roots are not given, as giving them would destroy the value of the problem.

1. $2 \pm 2\sqrt{-1}.$
 2. $1 \pm \sqrt{-1}; 3 \pm \sqrt{2}.$
 3. $\frac{3 \pm 3\sqrt{-3}}{2}.$
 4. $\pm 3\sqrt{-1}.$
 5. $\frac{-7 \pm \sqrt{-35}}{2}.$
 6. $\pm 3\sqrt{-1}.$
 8. $\pm 2\sqrt{-2}.$
 9. $-1 \pm \sqrt{-2}.$
 11. $2 \pm \sqrt{6}.$
 14. $\frac{2 - a \pm \sqrt{16 + 4a - 3a^2}}{2}.$
 15. $-1 \pm 2\sqrt{-1}.$
 16. $\frac{1 \pm \sqrt{-23}}{2}.$
 17. $\frac{-9 \pm \sqrt{-95}}{2}.$
 18. $\frac{-7 \pm \sqrt{-23}}{2}.$

Ex. 22. Page 106.

11. $1; -\frac{1}{2}.$
 12. $3; \frac{4}{3}.$
 13. $4; -\frac{2}{3}.$
 14. $7; 23.$

Ex. 23. Pages 108-111.

- | | |
|-------------------------------|------------------------------------|
| 1. 14; 15. | 15. \$360. |
| 2. 10; 11. | 16. 18 hours; 24 hours. |
| 3. 16; 17. | 17. 6 days. |
| 4. 12; 72. | 18. 5 hours; $7\frac{1}{2}$ hours. |
| 5. $\frac{5}{8}$. | 19. 45. |
| 6. $\frac{7}{8}$. | 20. 68. |
| 7. 10; \$2.40. | 21. 5 miles per hour; |
| 8. \$1.80; 20 days. | 1 mile per hour. |
| 9. 28 days. | 22. \$65 or \$35. |
| 10. 28 acres. | 23. \$120. |
| 11. 40 and 45; \$81 and \$72. | 24. 100; \$85. |
| 12. 75; \$4. | 25. 22; \$22. |
| 13. 60 yards by 40 yards. | 26. 9 hours; $6\frac{1}{4}$ hours. |
| 14. 25 yards by 48 yards. | 27. 120 miles. |

Ex. 24. Pages 116-119.

The upper values are the values of x , the lower values, the corresponding values of y .

- | | | |
|-----------------------------|--|--|
| 1. 5, 3;
3, 5. | 5. 6;
3. | 9. 3, $-7\frac{1}{2}$;
4, $\frac{1}{2}$. |
| 2. 9, -3 ;
-3 , 9. | 6. 5, $-4\frac{1}{2}$;
-3 , $3\frac{1}{2}$. | 10. 8, 6;
6, 8. |
| 3. 8, -3 ;
3, -8 . | 7. 2, $2\frac{1}{2}$;
3, $1\frac{1}{2}$. | 11. 4, $3\frac{1}{7}$;
-1 , $2\frac{1}{7}$. |
| 4. 10, 6;
-6 , -10 . | 8. 2, -2 ;
5, 7. | 12. 2, $-\frac{7}{8}$;
4, $-2\frac{7}{8}$. |

-
- | | |
|--|--|
| 13. 5, $-4\frac{1}{2}$;
3, $-3\frac{2}{15}$. | 26. ± 5 , $\pm 3\sqrt{-3}$;
∓ 1 , $\pm 2\sqrt{-3}$. |
| 14. 7, $3\frac{1}{5}$;
6, $1\frac{1}{5}$. | 27. ± 5 , $\pm 11\sqrt{-2}$;
± 2 , $\mp \frac{1}{2}\sqrt{-2}$. |
| 15. 6, 9;
9, 6. | 28. ± 4 , $\pm \frac{2}{\sqrt{7}}$;
∓ 3 , $\mp \frac{15}{\sqrt{7}}$. |
| 16. 9, -60 ;
12, $-22\frac{1}{2}$. | 29. ± 2 , $\pm \sqrt{2}$;
± 4 , $\pm 3\sqrt{2}$. |
| 17. 3;
-5 . | 30. ± 2 , $\pm \frac{7}{2\sqrt{2}}$;
∓ 4 , $\pm \frac{3}{2\sqrt{2}}$. |
| 18. 5, $\frac{1}{5}$;
-2 , $-\frac{2}{5}$. | 31. ± 2 , $\pm \frac{34}{\sqrt{19}}$;
± 4 , $\mp \frac{40}{\sqrt{19}}$. |
| 19. ± 9 , ± 5 ;
± 5 , ± 9 . | 32. ± 2 , $\pm \frac{4}{\sqrt{3}}$;
± 1 , $\pm \frac{1}{\sqrt{3}}$. |
| 20. ± 6 , ± 4 ;
∓ 4 , ∓ 6 . | 33. ± 5 , $\pm 4\frac{1}{2}$;
± 3 , $\pm 4\frac{1}{2}$. |
| 21. ± 3 , $\pm \frac{1}{\sqrt{2}}$;
± 2 , $\mp \frac{5}{\sqrt{2}}$. | 34. 4, 1;
1, 4. |
| 22. ± 3 , $\pm \frac{7}{\sqrt{2}}$;
± 1 , $\mp 4\sqrt{2}$. | 35. 5, -3 ;
3, -5 . |
| 23. ± 3 , $\pm 4\frac{1}{2}$;
± 2 , $\pm \frac{1}{2}$. | |
| 24. ± 6 ;
± 4 . | |
| 25. ± 4 , $\pm 4\frac{1}{2}$;
± 2 , $\pm \frac{1}{2}$. | |

36. 7, -4;
-4, 7.
37. 7, -5;
5, -7.
38. 5, 3;
3, 5.
39. 11, -3;
3, -11.
40. 4, 3;
3, 4.
41. 5, -3;
3, -5.
42. 3, -6, $\frac{-3 \pm \sqrt{33}}{2}$;
-6, 3, $\frac{-3 \mp \sqrt{33}}{2}$.
43. 4, 3, $\frac{-13 \pm \sqrt{793}}{14}$;
3, 4, $\frac{-13 \mp \sqrt{793}}{14}$.
44. 6, 3;
3, 6.
45. 4, -2;
2, -4.
46. 6, 3;
3, 6.
47. $3\frac{2}{3}$, $1\frac{1}{3}$, 0;
 $1\frac{1}{3}$, $3\frac{2}{3}$, 0.
48. 5, 2, 6, 1;
2, 5, 1, 6.
49. 3, 2, $\frac{-3 \pm \sqrt{17}}{2}$;
2, 3, $\frac{-3 \mp \sqrt{17}}{2}$.
50. 0, 3, 2;
0, 2, 3.
51. 7, 2, $-4 \pm \sqrt{19}$;
2, 7, $-4 \mp \sqrt{19}$.
52. 4, -3, $-19 \pm 6\sqrt{-3}$;
-3, 4, $-19 \mp 6\sqrt{-3}$.
53. 5, -3, $1 \pm \sqrt{-22}$;
-3, 5, $1 \mp \sqrt{-22}$.
54. 3, -2, $\frac{1 \pm 3\sqrt{-3}}{2}$;
2, -3, $\frac{-1 \pm 3\sqrt{-3}}{2}$.
55. 5, 3, $4 \pm 2\sqrt{-33}$;
3, 5, $4 \mp 2\sqrt{-33}$.
56. 4, 2, $\frac{3 \pm \sqrt{-55}}{8}$;
2, 4, $\frac{3 \mp \sqrt{-55}}{8}$.
57. 5, 3, $\frac{-5 \pm \sqrt{17}}{2}$;
3, 5, $\frac{-5 \mp \sqrt{17}}{2}$.
58. ± 6 , ± 6 ;
 ± 3 , ∓ 3 .

$$59. \pm 3, \pm 2; \\ \mp 2, \mp 3.$$

$$61. 8, 2; \\ 2, 8.$$

$$60. 5, -5, 3, -3; \\ 3, -3, 5, -5.$$

$$62. 27, 3; \\ 3, 27.$$

$$63. 3, -3, 3, -3, 2, -2, 2, -2; \\ 2, -2, -2, 2, 3, -3, -3, 3.$$

$$\frac{1 \pm \sqrt{-31}}{2}, \frac{-1 \pm \sqrt{-31}}{2}, \frac{1 \pm \sqrt{-31}}{2}, \frac{-1 \pm \sqrt{-31}}{2};$$

$$\frac{1 \mp \sqrt{-31}}{2}, \frac{-1 \mp \sqrt{-31}}{2}, \frac{-1 \pm \sqrt{-31}}{2}, \frac{1 \pm \sqrt{-31}}{2}$$

$$64. 3, -4\frac{1}{2}; \\ -2, 2\frac{1}{2}.$$

$$65. \frac{a}{2}; \\ \frac{b}{2}$$

$$66. \frac{a+2b}{2}, \frac{a-2b}{2}; \\ \frac{a-2b}{2}, \frac{a+2b}{2}.$$

$$67. 0, a+b, \frac{1}{2}[a-b \pm \sqrt{(a-b)(a+3b)}]; \\ 0, a+b, \frac{1}{2}[a-b \mp \sqrt{(a-b)(a+3b)}].$$

$$68. \pm \frac{a^2+b^2}{a-b}; \\ \pm \frac{2ab}{a-b}.$$

$$69. 3, 2, -3 \pm \sqrt{3}; \\ 2, 3, -3 \mp \sqrt{3}.$$

$$70. \pm 2, \pm 3\sqrt{\frac{2}{17}}, \pm 4, \pm 6\sqrt{\frac{2}{17}}; \\ \pm 4, \pm 6\sqrt{\frac{2}{17}}, \pm 2, \pm 3\sqrt{\frac{2}{17}}.$$

$$71. 1, 1, 4, -4; \\ 4, -4, 1, 1.$$

$$72. \pm \sqrt{\frac{a}{2b}}; \\ \pm \sqrt{\frac{b}{2a}}$$

$$73. \frac{a}{2}(\sqrt{-1} \pm \sqrt{3}), \quad \frac{a}{2}(-\sqrt{-1} \pm \sqrt{3});$$

$$\frac{a}{2}(\sqrt{-1} \mp \sqrt{3}), \quad \frac{a}{2}(-\sqrt{-1} \mp \sqrt{3}).$$

$$74. a \pm b, \quad -a \pm b;$$

$$a \mp b, \quad -a \mp b.$$

$$75. \pm(2a+b), \quad \pm(a+2b);$$

$$\pm(a+2b), \quad \pm(2a+b).$$

$$76. 0, \quad \frac{a+2 \pm \sqrt{a^2-4}}{2b};$$

$$0, \quad \frac{a+2 \mp \sqrt{a^2-4}}{2b}.$$

$$77. a+2b, \quad 2a+b, \quad \pm ab\sqrt{-1};$$

$$2a+b, \quad a+2b, \quad \mp ab\sqrt{-1}.$$

$$78. 6, \quad 1\frac{1}{2};$$

$$2, \quad 5\frac{1}{2};$$

$$3, \quad 3\frac{1}{2}.$$

$$81. 2, \quad -3;$$

$$4, \quad -5;$$

$$6, \quad -7.$$

$$79. 3, \quad -2\frac{1}{2};$$

$$4, \quad -2\frac{1}{2};$$

$$4, \quad -6\frac{1}{2}.$$

$$82. \frac{a^2}{a+b}, \quad -\frac{a^2}{a+b};$$

$$\frac{2ab}{a+b}, \quad -\frac{2ab}{a+b};$$

$$80. 8, \quad 2;$$

$$4, \quad 4;$$

$$2, \quad 8.$$

$$\frac{b^2}{a+b}, \quad -\frac{b^2}{a+b}.$$

Ex. 25. Pages 120-124.

$$1. 7 \text{ feet by } 5 \text{ feet.}$$

$$3. 3120 \text{ sq. in.}$$

$$2. 75 \text{ yards by } 16 \text{ yards.}$$

$$4. 1008 \text{ sq. in.}$$

$$5. 16 \text{ and } 10; \text{ or } -16 \text{ and } -10.$$

$$6. \frac{1}{2}(3 \pm \sqrt{5}) \text{ and } \frac{1}{2}(1 \pm \sqrt{5}).$$

- | | |
|---------------|---------------------------------------|
| 7. 7 and - 2. | 11. $\frac{2}{3}$ and $\frac{3}{4}$. |
| 8. 8 and 3. | 12. $\frac{2}{3}$, $\frac{4}{5}$. |
| 9. 48. | 13. 73 and 37. |
| 10. 347. | 14. 8 feet, 10 feet. |
15. 150 miles; 15 and 10 miles per day.
 16. 88 yards by 55 yards.
 17. 15 miles; 3, $2\frac{1}{4}$, 4 miles per hour.
 18. 49,500 lbs.; 20 oz.
 19. 72; or 45.
 20. 5 hours; 3 hours.
 21. A 36 pages, B 72 pages;
 or A 9 pages, B 45 pages.
 22. A 10, B 6; B works $1\frac{2}{3}$ hours;
 or A 8, B 12; B works $1\frac{1}{3}$ hours.
 23. 290 yards.
 24. 6 and $1\frac{1}{2}$ miles per hour.
 25. 13.
 26. 7%; 6%.
 27. 3500 at 6%; 1500 at 7%.
 28. 374; or 132. 29. 327. 30. 140 miles.

Ex. 26. Page 126.

1. 1, - 2, $\frac{-1 \pm \sqrt{-3}}{2}$, $1 \pm \sqrt{-3}$.
 2. ± 1 , ± 2 .
 3. 2, $-\sqrt[3]{12}$, $-1 \pm \sqrt{-3}$, $\sqrt[3]{12}\left(\frac{1 \pm \sqrt{-3}}{2}\right)$.
 4. ± 3 , $\pm \frac{1}{2}$.
 5. ± 1 , $\pm \sqrt{-1}$, $\pm \frac{1}{2}$, $\pm \frac{1}{2}\sqrt{-1}$.
 6. 1, $\frac{1}{2}$, and eight other values.

7. $-\sqrt[3]{2}, -\sqrt[3]{12}, \sqrt[3]{2}\left(\frac{1 \pm \sqrt{-3}}{2}\right), \sqrt[3]{12}\left(\frac{1 \pm \sqrt{-3}}{2}\right).$
8. $0, -\frac{1}{2}, \frac{1}{3}, \frac{1 \pm \sqrt{-3}}{4}, \frac{-1 \pm \sqrt{-3}}{6}.$
9. $\pm \sqrt[4]{21}, \pm \sqrt{-1}, \pm \sqrt[4]{21} \sqrt{-1}.$
10. $\sqrt[3]{-4}, \sqrt[3]{1}.$ 12. $\sqrt[3]{5}, \sqrt[3]{-8}.$
11. $\sqrt[2]{\frac{5}{2}}, \sqrt{-\frac{5}{2}}.$ 13. $\sqrt[3]{2a}, \sqrt[3]{-4a}.$
14. $\pm \frac{1}{\sqrt{6}}, \pm \frac{1}{\sqrt{-2}}.$
15. $\frac{1}{\sqrt[3]{4}}, -\frac{1}{\sqrt[3]{9}}, \frac{-1 \pm \sqrt{-3}}{2\sqrt[3]{4}}, \frac{1 \pm \sqrt{-3}}{2\sqrt[3]{9}}.$
16. $\pm \sqrt[4]{\frac{1}{14}}, \pm \sqrt[4]{-\frac{1}{14}}; \text{four other values}.$
17. $\pm \frac{3}{\sqrt{5}}; \pm \sqrt{-1}.$ 24. $64; (-\frac{2}{3})^6.$
18. $16; (-\frac{5}{4})^4.$ 25. $81; (-\frac{3}{4})^4.$
19. $729; (-\frac{27}{8})^3.$ 26. $\frac{1}{64}; (-\frac{3}{16})^6.$
20. $\sqrt[5]{81}; 2\sqrt[5]{8}.$ 27. $\frac{1}{128}; -\frac{8}{729}.$
21. $121; (-2)^2.$ 28. $\frac{64}{729}; -\frac{1}{27}.$
22. $512; -64.$ 29. $256; -\frac{729}{2}.$
23. $5\sqrt[3]{5}; \frac{7}{2}\sqrt[3]{\frac{7}{2}}.$ 30. $\frac{1}{8}; \frac{1}{2}.$

Ex. 27. Pages 129-131.

1. 5; 221. 7. 1; $\frac{1}{21}.$
2. 5; $1\frac{24}{121}.$ 8. 9; -7.
3. $\frac{25}{8}.$ 9. -4; $\frac{24}{17}.$
4. $\pm 5; \pm 3\sqrt{2}.$ 10. -2; $\frac{4}{3}.$
5. 0; -28. 11. $\frac{1}{2}; -2; 1\frac{1}{2}; -3.$
6. 3; 12. 12. 7; -4; 4; -1.

13. $1; -\frac{1}{2}; \frac{3 \pm \sqrt{41}}{12}$.
 14. $1\frac{1}{2}; 3\frac{1}{2}; 4; 1$.
 15. $2; \frac{1}{2}; \frac{21 \pm \sqrt{-71}}{12}$.
 16. $1; -\frac{4}{3}; \frac{-2 \pm \sqrt{313}}{12}$.
 17. $-1; 2\frac{1}{2}; \frac{3 \pm \sqrt{-15}}{4}$.
 18. $4; 1; 3; 2$.
 19. $5; -3; 1; 1$.
 20. $1\frac{1}{2}; -2; \frac{-2 \pm \sqrt{-62}}{6}$.
 21. $2; -\frac{1}{2}; \frac{3 \pm \sqrt{-39}}{4}$.
 22. 1.
 23. $\pm \sqrt{5}; \pm \sqrt{65}$.
 24. $1\frac{1}{4}$.
 25. ± 2 .
 26. $0; 2$.
 27. ± 2 .
 28. $\frac{2}{3}; 2\frac{8}{11}$.
 29. $4; -\frac{1}{4}$.
 30. $-2a$.
 31. $2; \frac{1}{2}$.
 32. $\pm 9\sqrt{2}$.
 33. $\frac{a(1 \pm \sqrt{5})}{2}$.
 34. $x = \pm \sqrt{\frac{a^4 - b^4}{7a^2 - 2b^2}}$.
 35. 4.
 36. ± 8 .
 37. $-1; +2; -1 \pm \sqrt{-3}, \frac{1}{2} \pm \frac{1}{2}\sqrt{-3}$.
 38. $4\frac{1}{2}$.
 39. $0; \pm \sqrt{3}$.
 40. ± 3 .
 41. $\frac{1 \pm \sqrt{1+4b^2}}{2a}$.
 42. $b \pm \sqrt{b^2 - ab}$.
 43. $0; \frac{9a}{16}$.
 44. $2, 6, \frac{-9 \pm \sqrt{33}}{2}$.
 45. $\frac{1}{2}(a+b+1) \pm \frac{1}{4}(\sqrt{4a+1} \pm \sqrt{4b+1})$.
 $\frac{1}{2}(a-b) \pm \frac{1}{4}(\sqrt{4a+1} \mp \sqrt{4b+1})$.
 46. $\frac{1}{4b} [\pm (a^2 + b^2) \pm \sqrt{10a^2b^2 - 3a^4 - 3b^4}]$.
 $\frac{1}{4b} [\pm (a^2 + b^2) \mp \sqrt{10a^2b^2 - 3a^4 - 3b^4}]$.

47. $\pm 4, \pm 8; \pm 9, \pm 18.$ 48. $0, 3 \pm 2\sqrt{2}; 0, (3 \pm 2\sqrt{2})^2.$ 49. $6, -4\frac{1}{2}; 12, 9.$

Ex. 28. Pages 135, 136.

1. $x^2 - 5x + 6 = 0.$ 4. $6x^2 - 7x + 2 = 0.$
 2. $x^2 + x - 20 = 0.$ 5. $12x^2 + 13x + 3 = 0.$
 3. $x^2 + 14x + 48 = 0.$ 6. $x^2 - 2ax + a^2 - 9b^2 = 0.$
 7. $9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0.$
 8. $x^2 - 4x + 1 = 0.$ 11. $3(x-7)(x+2).$
 9. $x^2 + 2x - 4 = 0.$ 12. $(3x-14)(3x+5).$
 10. $3x^2 - 6x + 1 = 0.$ 13. $(7x+6)(7x+1).$
 14. $(13x-2)(13x-2).$
 15. $\left(x - \frac{3 + \sqrt{-7}}{2}\right)\left(x - \frac{3 - \sqrt{-7}}{2}\right).$
 16. $\left(x + \frac{1 + \sqrt{-3}}{2}\right)\left(x + \frac{1 - \sqrt{-3}}{2}\right).$
 17. $(2x-7)(2x-7).$
 18. $(2x+3+2\sqrt{-1})(2x+3-2\sqrt{-1}).$
 19. 17. 23. $\frac{175}{64}.$ 27. $\frac{261}{64}.$
 20. 56. 24. $\frac{23}{7}.$ 28. $-\frac{p}{q}.$
 21. $\frac{33}{84}.$ 25. $\frac{175}{612}.$ 29. $-pq.$
 22. $\frac{23}{8}.$ 26. 833. 30. $3pq - p^3.$
 31. $p^2q - 2q^2.$ 33. $\frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$
 32. $p^4 - 4p^2q + 2q^2.$

34. When the sign of b is unlike that of a and c , and $b^2 - 4ac > 0$.

When a, b, c have like signs, and $b^2 - 4ac > 0$.

When a and c have unlike signs.

35. When $a^2c + ac^2 + b^3 = 3abc$. 36. When $b + c = 0$.

Ex. 29. Pages 141, 142.

1. $m = 1$ or $-\frac{1}{2}$; m between 1 and $-\frac{1}{2}$; m greater than 1 or less than $-\frac{1}{2}$.
2. $m = 3$ or $\frac{5}{4}$; m between 3 and $\frac{5}{4}$; m greater than 3 or less than $\frac{5}{4}$.
3. $m = 4$ or $-\frac{2}{3}$; m between 4 and $-\frac{2}{3}$; m greater than 4 or less than $-\frac{2}{3}$.
4. $m = 7$ or 23; m greater than 23 or less than 7; m between 7 and 23.
5. $m = 0$ or 4; m greater than 4 or less than 0; m between 0 and 4.
6. Min. value 4.
7. Min. value 7.
8. Max. value 7.
9. Min. value 4.
10. Min. value 16.
11. Min. value -32 .
12. Max. value $\frac{1}{24}$.
13. Max. value $\frac{225}{144}$.
14. Max. value $\frac{1}{2}$.
15. Max. value 1, min. value $-1\frac{2}{3}$.
16. Min. value -1 , max. value $+2$.
17. No max. or min.
18. Max. value $\frac{11}{2}$, min. value $\frac{1}{2}$.
19. Max. value 4, min. value 0.
20. Max. value 3, min. value $1\frac{1}{2}$.
21. Bisect the line.

22. Bisect the line. 23. Each part is a .
 24. The altitude of the rectangle is half the altitude of the triangle.
 25. The inscribed square. 26. The inscribed square.

Ex. 30. Page 145.

- | | |
|-----------------------------------|--|
| 1. $\pm (3 + \sqrt{5})$. | 11. $\pm (2\sqrt{3} - \sqrt{2})$. |
| 2. $\pm (2\sqrt{3} + \sqrt{5})$. | 12. $\pm (2\sqrt{5} - 3\sqrt{2})$. |
| 3. $\pm (\sqrt{7} + \sqrt{3})$. | 13. $\pm (3\sqrt{11} - 2)$. |
| 4. $\pm (\sqrt{11} + \sqrt{5})$. | 14. $\pm (3\sqrt{5} - 2\sqrt{3})$. |
| 5. $\pm (\sqrt{7} - \sqrt{2})$. | 15. $\pm (\frac{1}{2}\sqrt{10} - 1)$. |
| 6. $\pm 2(\sqrt{3} - \sqrt{2})$. | 16. $\pm (\sqrt{a+b} + \sqrt{a-b})$. |
| 7. $\pm (\sqrt{6} - \sqrt{3})$. | 17. $\pm (\sqrt{a^2 - b^2} - b)$. |
| 8. $\pm (7 - 3\sqrt{5})$. | 18. $\pm (3\sqrt{7} - 2\sqrt{6})$. |
| 9. $\pm (\sqrt{10} - \sqrt{3})$. | 19. $\pm (a - b - 2\sqrt{ab})$. |
| 10. $\pm (3 - \sqrt{2})$. | |

Ex. 31. Pages 151, 152.

- | | |
|--|-------------------------|
| 1. $-4; -72; \sqrt{15} \sqrt{-1}$. | |
| 2. $-\sqrt{\frac{1}{3}} \sqrt{-1}; 2; 3$. | |
| 3. $4 + 9\sqrt{-1}; 5 + 2\sqrt{6} \sqrt{-1}; -18 + 18\sqrt{3} \sqrt{-1}$. | |
| 4. 19. | 6. $19 + 19\sqrt{-3}$. |
| 5. 11. | 7. $65 - 13\sqrt{-2}$. |
| 8. $-6 - 26\sqrt{-15}$. | |
| 9. $\sqrt{ac} - ab \sqrt{bc} + (bc + a\sqrt{ab}) \sqrt{-1}$. | |
| 10. $6 - 4\sqrt{-1}; 12 + 2\sqrt{-7}$. | |

11. $\frac{3}{8} - \frac{1}{8}\sqrt{-1}$. 14. $\pm(4 - \sqrt{-6})$.
 12. $1 + 6\sqrt{-2}$. 15. $\pm(\sqrt{3} + 2\sqrt{-5})$.
 13. $\pm(2 + \sqrt{-3})$. 16. $\pm(5 - 3\sqrt{-7})$.
 22. $\pm(\sqrt{\frac{1}{2}} + \sqrt{-\frac{1}{2}}), \pm(\sqrt{\frac{1}{2}} - \sqrt{-\frac{1}{2}})$.
 23. $+1, -1, \pm(\frac{1}{2} + \frac{1}{2}\sqrt{-3}); \pm(\frac{1}{2} - \frac{1}{2}\sqrt{-3})$.
 24. $\pm 1, \pm\sqrt{-1}, \pm(\sqrt{\frac{1}{2}} + \sqrt{-\frac{1}{2}}); \pm\sqrt{\frac{1}{2}} - \sqrt{-\frac{1}{2}}$.
 25. $\frac{42}{288} - \frac{13}{878}\sqrt{-1}$.

Ex. 32. Page 154.

8. The first expression. 10. The first expression.
 9. The first expression. 11. The first expression.
 12. The first expression.

Ex. 33. Page 162-164.

2. 10 : 9. 18. $2\frac{2}{3}$.
 4. Greatest, 23 : 25 ;
 least, 3 : 4. 19. $\frac{125}{1944}$.
 5. The first. 20. $\frac{a-b}{c(a+b)}$.
 6. 4 : 15. 21. 7.
 7. 2 : 3. 22. $\sqrt{m^2 - 25}$.
 8. $a + x : a + 2x$. 31. 0.
 9. $x + 2 : 2x - 3$. 32. $a + b, \frac{a-b}{2}$.
 13. 6 and 15. 33. ± 2 .
 14. $\frac{mq - np}{p - q}$. 34. -1 .
 16. 150. 35. $\frac{bc - ad}{a - b - c + d}$.
 17. $1\frac{1}{2}$.

Ex. 34. Pages 168, 169.

1. 1240 rods \times 680 rods.
2. 4 : 1.
3. 5 : 9.
4. A's wages : B's wages :: 6 : 5.
5. 15 miles per hour ; 12 miles per hour.
6. 8 : 7.
8. 6 gals.
7. 2 : 1 or 1 : 2.
9. 10 gals.
10. 45 and 30 miles per hour.
11. A, \$200 ; B, \$150.
12. 70.
13. Last year the ratio was 3 : 7 ;
this year it is 45 : 104.

Ex. 35. Pages 174, 175.

1. $9\frac{3}{4}$.
2. $\frac{2}{3}$.
3. $41\frac{1}{2}$.
4. 5.
5. $3\frac{3}{14}$.
6. $64x^2 = 9y^3$.
7. $x^2y^3 = 108$.
8. 5.
9. 8.
10. $161\frac{1}{2}$ feet per second ; $402\frac{3}{4}$ feet.
11. $9\frac{1}{4}$.
12. $20\frac{1}{2}$ feet.
13. $516\frac{1}{2}$ ounces.
14. 87.1 days.
15. 1 inch.
16. Equal.
17. 10 inches.

Ex. 36. Pages 181-183.

1. 48.
2. - 9.
3. - 59.
4. $-4\frac{1}{2}$.
5. $-12\frac{3}{4}$.
6. 116.
7. - 55.
8. 228.
9. $\frac{29n - 5n^2}{12}$.
10. $\frac{59n - 5n^2}{24}$.
11. $d = 4\frac{1}{2}$; $s = 377$.
12. $d = \frac{3}{4}$; $s = 2757\frac{1}{4}$.
13. $d = 1\frac{1}{2}$; $l = 29\frac{1}{2}$.
14. $a = 0$; $d = 7$.
15. $a = 11$; $l = 88$.
16. $n = 16$; $l = 69$.
17. $a = 12$; $n = 14$.

18. 717 feet per second. 25. $2\frac{1}{4}$, $4\frac{1}{4}$, $6\frac{1}{4}$,
 19. $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$,, $11\frac{1}{2}$. 26. 5, 7, 9.
 20. 10th term. 27. 4, 11, 18.
 21. 9. 28. -3, 1, 5, 9.
 22. 6 and 14. 29. 10 terms.
 23. 105. 24. 47. 30. 2, 6, 10, 14.
 31. No; he would pay \$10.24 instead of 8.
 32. 234. 34. 3200 feet.
 33. 3825 feet. 35. 3, 6, 9, 12, 15.

Ex. 37. Pages 188-190.

1. 384. 8. $\frac{25}{8} - \frac{3^{12}}{8 \times 5^{10}}$.
 2. -4096.
 3. $-\frac{1}{3^{19}}$. 9. $\frac{75}{32} - \frac{3}{32 \times 5^{n-2}}$.
 4. $\frac{2^{17}}{3^{16}}$. 10. $2\frac{2}{3}$. 14. $\frac{17}{111}$.
 5. $(-\frac{4}{3})^{n-1}$. 11. $1\frac{1}{2}$. 15. $\frac{1223}{3300}$.
 6. 8188. 12. $\frac{5}{7}$. 16. $4\frac{19}{110}$.
 7. $\frac{27}{2} - \frac{1}{2 \times 3^{16}}$. 13. $\frac{3}{10}$. 17. $4\frac{853}{3330}$.
 18. $r = \frac{1}{2}$, $s = 69\frac{3}{4}$; $r = -\frac{1}{2}$, $s = 24\frac{3}{4}$.
 19. $a = 2$, $s = 254$. 25. 3, 192.
 20. $a = 5$, $l = 320$. 26. 3, 12, 48, 192.
 21. $a = 1$, $n = 5$. 27. 7, 14, 28, 56.
 22. 28, 56, 112. 28. 242.
 23. 6, 18, 54, 162, 486. 29. 1, 2, 3; 7, 2, -3.
 24. 5th. 30. 2, 5, 8; 26, 5, -16.
 31. 3, -9, 27, 63; 75, 15, 3, -9.

32. 1, 2, 4, 6.

33. 3, 27.

34. 139.

35. Any series with ratio $\frac{1}{2}$.

Ex. 38. Page 192.

1. $2\frac{2}{3}$, 3, 4, 6.2. 1, 9; $-7\frac{1}{3}$, $\frac{4}{3}$.

3. 12.

4. $13\frac{1}{3}$, $14\frac{2}{3}$,, $17\frac{1}{3}$, $18\frac{2}{3}$.

5. Sixth term.

7. 8, 2.

6. 6, $6\frac{2}{3}$, $6\frac{4}{3}$, $7\frac{5}{3}$.

8. 104, 234.

9. 4, 7, 28; -5, -11, 55.

Ex. 39. Pages 196-198.

1.

x	y
12	0
11	1
....
0	12

2.

x	y
36	1
25	3
14	5
3	7

3.

x	y
11	1
2	5

4.

x	y
8	2
3	10

5.

x	y
18	5
15	10
....
0	35

6.

x	y
116	1
96	4
....
16	16

7.

x	y
36	0
35	3
....
0	108

8.

x	y
125	4
110	12
....
5	68

9. $x=2$
 $y=1$

10. $x=4$
 $y=3$

11. $x = 7,$

$y = 1.$

12. $x = 8,$

$y = 3.$

13. $x = 7,$

$y = 2.$

14. $x = 11,$

$y = 11.$

15.

150, 5;

133, 12;

....

14, 61.

18.

81, 8;

57, 32;

33, 56;

9, 80.

22.

$\frac{1}{24}, \frac{1}{16};$

$\frac{1}{24}, \frac{1}{16};$

....

$\frac{1}{24}, \frac{1}{16}.$

16.

14, 6;

31, 14;

48, 22;

....

19.

275, 39;

132, 182.

23.

\$5, \$2.

1, 41;

3, 36;

....

17, 1.

20. 39.

17.

6, 2;

21, 9;

36, 16;

....

21.

21, 172;

109, 260;

....

24.

Calves. Figs.

3, 33;

6, 28;

....

21, 3.

25. 3 pigs, 21 geese, 16 chickens;
or 6 pigs, 2 geese, 32 chickens.

26.

$x \quad y$
 $5\frac{5}{18} \quad 5\frac{5}{18}$

$8\frac{1}{2} \quad 17\frac{1}{2}$

$23\frac{1}{2} \quad 70$

....

27.

$x \quad y$
 $\frac{1}{4} \quad 1\frac{3}{4}$

28.

6, 56, 8;

12, 42, 16;

18, 28, 24;

....

24, 14, 32.

29. 3 ways (or 4 ways).

31. $42m + 5$.

30. 5 ways.

32. $504m - 1$.

33. 19 oxen, 1 sheep, 80 hens.

34. Calves \$7, lambs \$4, pigs \$3.

Ex. 40. Pages 205, 206.

1. $1 + 15x + 90x^2 + 270x^3 + 405x^4 + 243x^5$.

2. $1 + \frac{8x}{3} + \frac{8x^2}{3} + \frac{32x^3}{27} + \frac{16x^4}{81}$.

3. $1 - \frac{4x^{\frac{1}{2}}}{3} + \frac{2x^3}{3} - \frac{4x^{\frac{3}{2}}}{27} + \frac{x^6}{81}$.

4. $64 + 192x^2 + 240x^4 + 160x^6 + 60x^8 + 12x^{10} + x^{12}$.

5. $\frac{32}{x^5} - \frac{20}{x^2} + 5x - \frac{5x^4}{8} + \frac{5x^7}{128} - \frac{x^{10}}{1024}$.

6. $\frac{32a^5}{x^5} - \frac{20a^2}{x^2} + \frac{5x}{a} - \frac{5x^4}{8a^4} + \frac{5x^7}{128a^7} - \frac{x^{10}}{1024a^{10}}$.

7. $729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$.

8. $\frac{32x^{10}}{y^5} - \frac{20x^8}{y^{\frac{10}{3}}} + \frac{5x^6}{y^{\frac{5}{3}}} - \frac{5x^4}{8} + \frac{5x^2y^{\frac{5}{3}}}{128} - \frac{y^{\frac{10}{3}}}{1024}$.

9. $\frac{a^6}{b^4} - \frac{a^{\frac{7}{2}}}{b^{\frac{3}{2}}} + \frac{3a}{8b^{\frac{1}{2}}} - \frac{b^{\frac{5}{2}}}{16a^{\frac{3}{2}}} + \frac{b^3}{256a^4}$.

10. $1 + 16x + 108x^2 + 400x^3 + 886x^4 + 1200x^5 + 972x^6 + 432x^7 + 81x^8$.

11. $a^6 - 3a^5x - 3a^4x^2 + 11a^3x^3 + 6a^2x^4 - 12ax^5 - 8x^6$.

12. $7x^2$. 13. $-\frac{15}{256x^{14}}$.

14. $-\frac{91x^{\frac{5}{2}}}{4^{10}}$.

$$15. -\frac{8855 \times 2^{19}}{3^{19} \sqrt{x^3}}.$$

$$16. -\frac{595}{2048 \sqrt{x^{25}}}.$$

$$17. \frac{8 \times 7 \times 6 \cdots (9-r)}{1 \times 2 \times 3 \cdots r} \sqrt[3]{\frac{3^r}{2^r}} \sqrt{x^{24-5r}}.$$

$$18. (-1)^r \frac{10 \times 9 \times 8 \cdots (11-r)}{1 \times 2 \times 3 \cdots r} \frac{x^{r-5}}{3^{\frac{10-r}{2}} 2^r}.$$

$$19. (-1)^{r+2} \frac{12 \times 11 \times 10 \cdots (11-r)}{1 \times 2 \times 3 \cdots (r+2)} \frac{x^{\frac{12-3r}{2}} y^{2r-8}}{3^{\frac{r+2}{2}} 2^{10-r}}.$$

$$20. \frac{168399 x^3}{8192}.$$

$$21. -\frac{173745 \sqrt{3a^3}}{2048 \sqrt{x}}; \quad \frac{521235 a^3 \sqrt{x}}{2048 \sqrt{2}}.$$

$$22. (-1)^r \frac{11 \times 10 \times 9 \cdots (13-r)}{1 \times 2 \times 3 \cdots (r-1)} \frac{x^{\frac{104-5r}{6}}}{2^{\frac{3r+8}{2}}}.$$

$$25. x^4 - \frac{3}{2} + \frac{1}{16x^4} + \left(2x^2 - \frac{1}{2x^2}\right) \sqrt{-1};$$

$$\left(1 - \frac{5}{4}x^{\frac{1}{2}} + \frac{5}{8}x^{\frac{3}{2}} - \frac{5}{32}x + \frac{5}{256}x^{\frac{5}{2}} - \frac{x^{\frac{7}{2}}}{1024}\right) \sqrt{-1};$$

$$\left(-\frac{a^{\frac{5}{2}}}{32} - \frac{5a}{16} + \frac{5}{4a^{\frac{1}{2}}} - \frac{5}{2a^2} + \frac{5}{2a^{\frac{3}{2}}} - \frac{1}{a^5}\right) \sqrt{-1}.$$

Ex. 41. Pages 213, 214.

$$1. 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots. \quad 2. 1 + \frac{x}{4} - \frac{3x^2}{32} + \frac{7x^3}{128} - \cdots.$$

$$3. 1 + \frac{x}{3} + \frac{2x^2}{9} + \frac{14x^3}{81} + \cdots.$$

$$4. 1 + 4x + 10x^2 + 20x^3 + \cdots.$$

$$5. 1 + \frac{5x}{2} + \frac{15x^2}{8} + \frac{5x^3}{16} - \dots.$$

$$6. \frac{1}{a^{\frac{1}{2}}} + \frac{x^2}{4a^{\frac{3}{2}}} + \frac{5x^4}{32a^{\frac{5}{2}}} + \frac{15x^6}{128a^{\frac{7}{2}}} + \dots.$$

$$7. 2^{\frac{1}{2}} - \frac{3 \times 2^{\frac{1}{2}}}{10}x - \frac{9 \times 2^{\frac{1}{2}}}{50}x^2 - \frac{81 \times 2^{\frac{1}{2}}}{500}x^3 - \dots.$$

$$8. 2^{\frac{2}{3}} \left(1 - \frac{x^2}{3} - \frac{x^4}{36} - \frac{x^6}{162} - \dots \right).$$

$$9. 1 - \frac{3x^2}{2} + \frac{21x^4}{8} - \frac{77x^6}{16} + \dots.$$

$$10. -\frac{33x^7}{16}.$$

$$11. \frac{559130x^9}{81a^{\frac{22}{3}}}.$$

$$12. (-1)^{r-1} \frac{2 \times 5 \times 8 \dots (3r-4)x^r}{1 \times 2 \times 3 \dots r \times 3^r a^{\frac{3r-1}{3}}}.$$

$$13. \frac{5 \times 7 \times 9 \dots (2r+3) \times 2^r x^{2r}}{1 \times 2 \times 3 \dots r \times a^{2r+5}}.$$

$$14. 8.06224.$$

$$15. 2.002224.$$

$$16. 1 + \frac{4x}{5} - \frac{2x^2}{25} - \frac{196x^3}{125} - \dots.$$

$$17. 351.$$

$$19. -\frac{22x^5}{729}.$$

$$20. 1 + x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{3x^4}{8} + \frac{3x^5}{8}.$$

$$23. r \text{ even:}$$

$$\frac{1 \times 3 \times 5 \dots 3r-1}{2 \times 4 \times 6 \dots 3r}.$$

$$r \text{ odd:}$$

$$-\frac{1 \times 3 \times 5 \dots 3r-2}{2 \times 4 \times 6 \dots 3r-1}.$$

Ex. 42. Pages 222, 223.

1. 0.7781.	18. 1.3980.	35. $\bar{1}.6320$.
2. 1.1761.	19. 3.3804.	36. 0.2219.
3. 1.3222.	20. 3.4950.	37. 0.3980.
4. 1.1461.	21. 0.1003.	38. 0.3680.
5. 1.3980.	22. 0.3495.	39. 0.5441.
6. 1.4771.	23. 0.1398.	40. $\bar{2}.2219$.
7. 1.6232.	24. 0.0768.	41. $\bar{3}.3980$.
8. 2.6232.	25. 0.2258.	42. $\bar{2}.1461$.
9. $\bar{2}.3222$.	26. 0.2415.	43. 1.8539.
10. $\bar{1}.5441$.	27. 1.1650.	44. 1.2219.
11. $\bar{3}.5441$.	28. 0.3904.	45. $\bar{1}.5441$.
12. $\bar{3}.6020$.	29. 2.4465.	46. 0.4559.
13. 0.3222.	30. 0.4730.	47. $\bar{2}.8539$.
14. 1.2040.	31. 0.5243.	48. $\bar{5}.1707$.
15. $\bar{2}.7481$.	32. $\bar{1}.6020$.	49. 4.8293.
16. $\bar{1}.7993$.	33. $\bar{1}.4559$.	50. 5.9333.
17. 0.9030.	34. $\bar{1}.7781$.	51. 4.1040.

Ex. 43. Page 229.

1. 1.7782.	7. 4.8490.	13. 17,730.
2. 2.0043.	8. 4.0802.	14. 4295.
3. 2.9996.	9. $\bar{3}.9943$.	15. 53,910.
4. 3.5775.	10. $\bar{4}.6554$.	16. 93.98.
5. 3.7350.	11. $\bar{2}.1277$.	17. 0.1099.
6. 3.9581.	12. $\bar{1}.2983$.	18. 0.695.

Ex. 44. Pages 230, 231.

- | | | |
|---------------|----------------|---------------|
| 1. 41.64. | 12. - 5.218. | 23. 1.938. |
| 2. 0.02989. | 13. 2.257. | 24. 1.568. |
| 3. 0.3131. | 14. 15.78. | 25. 0.002431. |
| 4. 8.292. | 15. 1.615. | 26. 33.04. |
| 5. - 28.47. | 16. 7.212. | 27. 0.5096. |
| 6. - 0.3144. | 17. 0.0001174. | 28. 1.54+. |
| 7. 0.0104. | 18. 5.487. | 29. 2.66+. |
| 8. - 660,600. | 19. 2.381. | 30. 1.65+. |
| 9. 1.308. | 20. 2.154. | 31. 7.53-. |
| 10. - 111.3. | 21. 1.292. | 32. 2.12+. |
| 11. 0.001086. | 22. 146.8. | 33. 0.45+. |

Ex. 45. Page 232.

- | | | |
|------------|----------------|---------------------------------|
| 1. 0.6931. | 8. - (2.3742). | 15. 1.356. |
| 2. 1.0986. | 9. 2.807. | 16. 2.873. |
| 3. 4.6052. | 10. 1.262. | 17. - (1.262). |
| 4. 3.4864. | 11. 1.585. | 18. 0.6523. |
| 5. 2.0657. | 12. 1.209. | 19. $16\sqrt{2}$. |
| 6. 0.2070. | 13. 0.9464. | 20. $\frac{3}{4}\sqrt[3]{18}$. |
| 7. 0.6935. | 14. 0.7743. | |

Ex. 46. Pages 241, 242.

- | | |
|--|---------------------------|
| 1. Between 48 and 49. | 4. $4.310 \times A$. |
| 2. (1) $\frac{B-A}{Ar}$; | 5. \$444, nearly. |
| (2) $\frac{\log B - \log A}{\log R}$. | 6. \$20,270, nearly. |
| 3. \$0.00366. | 7. 5.35 per cent, nearly. |
| | 8. \$34.40. |

- | | |
|---------------------------|------------------|
| 9. 22. | 14. 4 per cent |
| 10. 115, nearly. | 15. \$7377. |
| 11. 5 per cent, nearly. | 16. A little ov |
| 12. 4.2 per cent, nearly. | 17. A little ov |
| 13. \$32.30. | 18. \$969, neari |

Ex. 47. Pages 257-259.

- | | |
|---|---|
| 1. 120. | 13. $\frac{\overline{m} \overline{n}}{\overline{r} \overline{m-r} \overline{s} \overline{s}}$ |
| 2. 180. | 14. 739,200. |
| 3. 1008 ; 224. | 15. $3780 \times \overline{9}$. |
| 4. 15,600. | 16. 205,200. |
| 5. 6. | 17. 173,641,000. |
| 6. 120. | 18. 5040 ; 144. |
| 7. 5040. | 19. 1080. |
| 8. 30,240. | 20. 1728. |
| 9. $\frac{\overline{60}}{\overline{8} \overline{52}}$ | 21. 432. |
| 10. 27,405 nights ;
3654 times. | 22. 109,600. |
| 11. 400,400. | 24. 10. |
| 12. $\frac{\overline{14} \overline{26}}{\overline{6} \overline{8} \overline{10} \overline{16}}$ | 25. $n = 11 ; r = 3$. |
| | 26. 52. |
| | 27. 455. |

Ex. 48. Pages 265, 266.

- | | | |
|----------|--------------------------------|---------|
| 1. 216. | 4. $\frac{\overline{11}}{8}$. | 7. 312. |
| 2. 63. | 5. 2520. | 8. 7. |
| 3. 7920. | 6. 420. | 9. 66. |

10. $\frac{n(n+1)(n+2)}{6}$

12. $n-1-i$

11. 325.

13. 63; 32,768

16. $(m+1)(n+1)(p+1)-2$ divisors besides 1 and the number itself.

$\frac{13}{3n+8}; \frac{13}{240}$

Ex. 49. Pages 271, 272.

$\frac{1}{2}; \frac{1}{4}$

*8. $\frac{5}{6} - \frac{1}{(n+2)(n+3)}; \frac{5}{6}$

Ex. 60. Pages 340, 341.

1. $\frac{3}{x+4} + \frac{4}{x-5}$

3. $\frac{8}{3(x-5)} - \frac{1}{3(2x-1)}$

2. $\frac{6}{x+3} - \frac{6}{x+4}$

4. $\frac{4}{7(x+2)} + \frac{3}{7(x-5)}$

5. $\frac{1}{x-1} - \frac{x+2}{x+1}$

$\frac{3}{3}$

Ex. 53. Pages 304, 305.

1. 2220.

7. 152.

13. 22.26.

2. 12,431.

8. 342.

14. 0.0302.

$3(x-1) \cdot (x-1)^2 \cdot 3(x+2)$

9. $\frac{10x-7}{3(x^2+x+1)} - \frac{4}{3(x-1)}$

10. $\frac{5}{2^x-1} - \frac{4}{3^x-1}$

19. Scale of 3. 21. 1, 8, 16, 64, 256 lbs.

Ex. 54. Page 312.

1. 105.

2. 3.

3. 6.

4. 42.

Ex. 51. Pages 287, 288.

1. $\frac{1}{8}$. 2. $\frac{1}{2}$. 3. $\frac{3}{5}$. 4. $\frac{15}{17}$. 5. $\frac{3}{8}$.
 6. \$0.75; \$3. 7. $\frac{9999}{10000}$. 8. $\frac{11}{50}$. 9. $\frac{13}{18}$.

Ex. 52. Pages 298, 299.

1. $\frac{7}{80}$; $\frac{22}{51}$; $\frac{73}{99}$.
 2. 180.
 3. 1008; 224.
 4. 15,600.
 5. 6.
 6. 120.
 7. 5040.
 8. 30,240.
 9. $\frac{60}{8 \mid 52}$
 10. 11,040.
 11. 11,040.
 12. $\frac{14 \mid 20}{6 \mid 8 \mid 10 \mid 16}$
 13. $\frac{r \mid m-r \mid s \mid n-s}{r \mid m-r \mid s \mid n-s}$
 14. 739,200.
 15. 3780×9 .
 16. 205,200.
 17. 173,641,000.
 18. 5040; 144.
 19. 1080.
 20. 1728.
 21. 432.
 22. 4970.

Ex. 59. Page 338.

12. $\frac{14 \mid 20}{6 \mid 8 \mid 10 \mid 16}$
 27. 455.

Ex. 48. Pages 265, 266.

2. $\frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)}; \frac{3}{4}$
 3. $\frac{1}{4} - \frac{1}{4(4n+1)}; \frac{1}{4}$

$$4. \frac{3}{5} - \frac{6}{5(5n+2)}; \frac{3}{5}$$

$$5. \frac{13}{240} - \frac{6n+13}{6(3n+5)(3n+8)}; \frac{13}{240}$$

$$6. \frac{n}{12(n+1)}; \frac{1}{12}$$

$$*7. \frac{1}{4} - \frac{1}{2(n+1)(n+2)}; \frac{1}{4}$$

$$*8. \frac{5}{6} - \frac{3n+5}{(n+2)(n+3)}; \frac{5}{6}$$

Ex. 60. Pages 340, 341.

$$1. \frac{3}{x+4} + \frac{4}{x-5}$$

$$3. \frac{8}{3(x-5)} - \frac{1}{3(2x-1)}$$

$$2. \frac{6}{x+3} - \frac{6}{x+4}$$

$$4. \frac{4}{7(x+2)} + \frac{3}{7(x-5)}$$

$$5. \frac{1}{x-1} - \frac{x+2}{x^2+x+1}$$

$$6. \frac{3}{4x} - \frac{1}{8(x-2)} + \frac{3}{8(x+2)}$$

$$7. \frac{4}{25x} - \frac{4}{5x^2} + \frac{71}{25(x+5)}$$

$$8. \frac{11}{3(x-1)} + \frac{2}{(x-1)^2} + \frac{10}{3(x+2)}$$

$$9. \frac{10x-7}{3(x^2+x+1)} - \frac{4}{3(x-1)}$$

$$10. \frac{5}{2x-1} - \frac{4}{3x-1}$$

$$11. \frac{7}{3x-5} - \frac{5}{4x+3}$$

10. 2749.

12. 1.2736.

Ex. 61. Pages 345, 346.

1. $1 + 2x + 4x^2 + 8x^3 + \dots$.

2. $\frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8} + \frac{27x^3}{16} + \dots$.

3. $\frac{1}{2} - \frac{x}{4} + \frac{3x^2}{8} - \frac{9x^3}{16} + \dots$.

4. $1 - 2x + x^2 + x^3 + \dots$.

5. $5 - 7x + 12x^2 - 19x^3 + \dots$.

6. $4x + 2x^2 - 8x^3 - 22x^4 - \dots$.

7. $-x + 2x^2 - x^3 - x^5 + \dots$.

8. $-\frac{1}{x^2} + \frac{1}{x} - 2 + x + \dots$.

9. $-\frac{1}{x} + 2x + x^2 - 2x^4 - \dots$.

10. $\frac{4}{x} - \frac{8}{x^2} + \frac{16}{x^3} - \frac{32}{x^4} + \dots$.

11. $-1 + \frac{5}{x} - \frac{15}{x^2} + \frac{45}{x^3} - \dots$.

12. $\frac{2}{x} + \frac{1}{x^2} + \frac{5}{x^3} + \frac{16}{x^4} + \dots$.

13. $1 + \frac{1}{x} + \frac{3}{x^2} + \frac{6}{x^3} + \dots$.

14. $\frac{3}{x^2} + \frac{4}{x^3} + \frac{5}{x^4} + \frac{6}{x^5} + \dots$.

15. $\frac{1}{x} + \frac{1}{x^4} + \frac{1}{x^5} + \frac{1}{x^6} + \dots$.

16. $\frac{1}{4} - \frac{1}{4(4n+1)}; \frac{1}{4}$.

12. $x = 1; y = 2; z = -1; w = 3.$

13. $x = 2; y = 4; z = -1; w = -3.$

14. $5x^2 + 9x + 1 = 0.$

15. $1 - 6x + 9x^2 + 2x^4 - 5x^5 = 0.$

16. $a^3 + b^3 + c^3 - 3abc = 0.$

17. $(ab' - a'b)(bc' - b'c) = (ac' - a'c)^2.$

18.
$$\begin{vmatrix} a & b & c \\ b^2 - ac + a^2q & bc + a^2r & 0 \\ 0 & b^2 - ac + a^2q & bc + a^2r \end{vmatrix} = 0.$$

Ex. 67. Page 383.

3. $-a(a-b)(b-c)(c-d).$

4. Multiply both sides of the equation to be proved by

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Ex. 68. Page 390.

1. $x^3 - x^2 - 3x - 4; -9.$

2. $x^3 + 3x^2 + 6x + 20; +53.$

3. $2x^3 + 7x^2 + 6x + 5; 0.$

4. $3x^3 - 9x^2 + 29x - 93; +329.$

5. $ax^2 - (ah - 3b)x + (ah^2 - 3bh + 3c);$
 $-ah^3 + 3bh^2 - 3ch + d.$

9. $-44.$

11. $-1553.$

10. $2749.$

12. $1.2736.$

Ex. 69. Page 394.

15. $x^3 - x^2 - 44x + 84 = 0$.
16. $x^3 - 3x^2 - 10x + 24 = 0$.
17. $x^3 - 4x = 0$.
18. $x^4 - 5x^2 + 4 = 0$.
19. $x^3 - 11x^2 + 40x - 50 = 0$.
20. $4x^4 - 16x^3 + 15x^2 + 4x - 4 = 0$.
21. $x^5 + 6x^4 - 13x^3 - 78x^2 + 36x + 216 = 0$.
22. $36x^4 + 36x^3 - 37x^2 - 11x + 6 = 0$.
23. $x^4 - 10x^3 + 32x^2 - 34x + 7 = 0$.
24. $200x^3 + 15x^2 - 21x + 10 = 0$.
25. $5000x^4 + 5750x^3 - 625x^2 - 405x - 18 = 0$.
26. $x^4 - 6x^3 + 18x^2 - 30x + 25 = 0$.

Ex. 70. Page 400.

1. $x^3 - 3x^2 - 10x + 24 = 0$; $x^3 + 3x^2 - 10x - 24 = 0$.
2. 17. 4. 74. 6. -14. 8. 317. 10. 12.
3. 11. 5. 15. 7. 53. 9. 51. 11. $p^2 - 2q$.
12. $3r - pq$. 14. $q^2 - 2pr$.
13. $-p^3 + 3pq - 3r$. 15. $p^4 - 4p^2q + 2q^2 + 4pr$.
16. $r - pq$. 17. $2p - \frac{q^2}{r}$. 18. $\frac{pq}{r} - 3$.
19. $\frac{2p^3q - 4pr - 2q^2}{r - pq}$. 20. $pq = r$.
21. $36p^3r - 8p^2q^2 - 182pqr - 36q^3 + 243r^2 = 0$.
22. $2p^3 - 9pq + 27r = 0$.
23. $27r^3 - 9pqr + 2q^3 = 0$.

Ex. 72. Page 409.

1. $2x$. 3. $-\frac{1}{x^2}$. 5. $4x^3$. 7. $-\frac{4}{x^5}$.
2. $3x^3$. 4. $-\frac{2}{x^3}$. 6. $-\frac{3}{x^4}$. 8. $2x + 1$.
9. $3x^2 + 4x$. 10. $2x + 2a$.
11. $-\frac{2x}{(x^2 - 3)^2}$. 12. $-\frac{2}{(x + 1)^2}$.

Ex. 73. Page 411.

9. $2x + 1$. 11. $(x - 3)(3x + 5)$.
10. $3x^2 - 14x + 11$. 12. $(x - 4)(4x^2 - 11x)$.
13. $2(x - \alpha)(x - \beta)(2x - \alpha - \beta)$.
14. $3x^2 - 2(\alpha + \beta + \gamma)x + \beta\gamma + \gamma\alpha + \alpha\beta$.
15. $4x^3 + 12x^2 - 38x - 46$.
16. $4x^3 - 12x^2 + 20x - 8$.

Ex. 74. Page 414.

5. $3ax^2 + 6bx + 3c$;
 $6ax + 6b$;
 $6a$.
6. $4ax^3 + 12bx^2 + 12cx + 4d$;
 $12ax^2 + 24bx + 12c$;
 $24ax + 24b$;
 $24a$.
7. $3x^2 - 2(2\alpha + \beta)x + \alpha^2 + 2\alpha\beta$;
 $6x - 4\alpha - 2\beta$;
 6 .
8. $3x^2 - 2(\alpha + \beta + \gamma)x + \beta\gamma + \gamma\alpha + \alpha\beta$;
 $6x - 2(\alpha + \beta + \gamma)$;
 6 .

9. $4x^3 - 6(\alpha + \beta)x^2 + 2(\alpha^2 + 4\alpha\beta + \beta^2)x - 2\alpha\beta(\alpha + \beta);$
 $12x^2 - 12(\alpha + \beta)x + 2(\alpha^2 + 4\alpha\beta + \beta^2);$
 $24x - 12(\alpha + \beta);$
 24.

Ex. 76. Page 419.

1. $y^3 + 3y^2 + 4y - 2.$
2. $y^4 + 16y^3 + 94y^2 + 246y + 245.$
3. $3y^4 + 34y^3 + 146y^2 + 281y + 200.$
4. $2y^4 - 19y^3 + 72y^2 - 131y + 86.$
5. $2y^4 - 26y^3 + 130y^2 - 299y + 263.$

Ex. 77. Pages 426, 427.

1. $x^3 + 3x^2 + 2x + 4 = 0.$
2. $x^4 + 12x^3 + 16x - 16 = 0.$
3. $2x^4 + 9x^3 + 9x^2 + 162x - 324 = 0.$
4. $x^4 - 6x^3 - 24x - 64 = 0.$
5. $3x^5 - 16x^3 - 32x - 224 = 0.$
6. $y^3 - 2y^2 + 18y + 18 = 0.$
7. $y^3 + 10y^2 - 42y + 576 = 0.$
8. $y^4 + 5y^3 - 40y^2 + 2500y - 30,000 = 0.$
9. $y^5 + 3y^4 + 24y^3 - 72y^2 + 1296y - 23,328 = 0.$
13. $y^3 - 8y^2 + 12y + 9 = 0.$
14. $y^4 + 2y^3 - 8y^2 - 6y + 15 = 0.$
15. $y^3 + 4y^2 - 15y - 18 = 0.$
16. $y^4 + 17y^3 + 92y^2 + 172y + 96 = 0.$
17. $y^4 + 1.2y^3 + 1.54y^2 - 2.292y + 3.1981 = 0.$
18. $y^4 - 4.6y^3 + 3.56y^2 + 3.104y - 6.5424 = 0.$

$$19. y^3 + 2y^2 - 11y - 25 = 0.$$

$$20. y^3 - 20y^2 + 100y - 547 = 0.$$

$$21. y^3 + 17y^2 + 75y - 29 = 0.$$

Ex. 82. Page 449.

1. 1.154⁺.

3. 4.264⁺.

2. 3.135⁻.

4. 2.833⁺.

5. -4.532⁺.

6. -1.064⁺.

7. -0.732⁺.

Ex. 83. Pages 455, 456.

1. 1.879385.

3. 1.580947⁺.

2. 3.263389⁺.

4. 2.656117.

5. 0.593686; 2.047276.

6. 2.381966; 4.618034.

7. 32.865384.

10. 43.

8. 624.

11. -2.469546.

9. 29.

12. -2.768345.

13. 2.356896; 2.692021.

14. 2.121320; 2.123113.

15. 3.213128; 3.229521.

Ex. 84. Page 462.

1. $[3, 4]$; $[4, 5]$; $[-3, -4]$.

2. $[4, 5]$. Two imaginary roots.

3. Two roots between 0 and -1;
two roots between 2 and 3.

4. Two real roots, one positive and one negative.

5. Roots all imaginary.
6. Two real roots.
7. Two imaginary roots.
8. Four imaginary roots.

Ex. 85. Page 467.

1. $\pm 1; \frac{-7 \pm 3\sqrt{5}}{2}$.
2. $\frac{-1 + \sqrt{2} \pm \sqrt{-1 - 2\sqrt{2}}}{2};$
 $\frac{-1 - \sqrt{2} \pm \sqrt{-1 + 2\sqrt{2}}}{2}$.
3. $\pm 1; \frac{3 + \sqrt{-7} \pm \sqrt{-14 + 6\sqrt{-7}}}{4};$
 $\frac{3 - \sqrt{-7} \pm \sqrt{-14 - 6\sqrt{-7}}}{4}$.
4. $2 \pm \sqrt{3}; \frac{1 \pm \sqrt{-3}}{2}$.
5. $1; 1; \frac{1 \pm \sqrt{-15}}{4}$.
6. $-1; 2 \pm \sqrt{3}; \frac{1 \pm \sqrt{-3}}{2}$.
7. $3 \pm 2\sqrt{2}; 2 \pm \sqrt{3}$.
8. $-1; \frac{1 - m \pm \sqrt{m^2 - 2m - 3}}{2}$.
9. $-1; \frac{\sqrt{3} \pm \sqrt{-1}}{2}; \frac{-\sqrt{3} \pm \sqrt{-1}}{2}$.
10. $1; \pm \sqrt{-1}; \frac{-1 \pm \sqrt{-35}}{6}$.

1.
2.
3.

Ex. 86. Page 469.

$$1. \pm \sqrt{-1}; \frac{\sqrt{3} \pm \sqrt{-1}}{2}; \frac{-\sqrt{3} \pm \sqrt{-1}}{2}.$$

$$2. \pm 1; \pm \sqrt{-1}; \sqrt{\frac{1}{2}} \pm \sqrt{-\frac{1}{2}}; -\sqrt{\frac{1}{2}} \pm \sqrt{-\frac{1}{2}}.$$

$$3. 1; \frac{-1 \pm \sqrt{-3}}{2}; \sqrt[3]{\frac{-1 \pm \sqrt{-3}}{2}}.$$

$$4. 3; \frac{-3 + 3\sqrt{5} \pm 3\sqrt{-10 - 2\sqrt{5}}}{4};$$

$$\frac{-3 - 3\sqrt{5} \pm 3\sqrt{-10 + 2\sqrt{5}}}{4}.$$

$$5. z^5 + z^4 - 4z^3 - 3z^2 + 3z + 1 = 0.$$

Ex. 87. Page 473.

$$1. -2 + 2\sqrt[3]{2} + \sqrt[3]{4}; -2 + 2\omega\sqrt[3]{2} + \omega^2\sqrt[3]{4};$$

$$-2 + 2\omega^2\sqrt[3]{2} + \omega\sqrt[3]{4}.$$

$$2. \frac{2 + \sqrt[3]{2} + 2\sqrt[3]{4}}{3}; \text{etc.}$$

$$3. 1 + \sqrt[3]{3} + \sqrt[3]{9}; \text{etc.}$$

$$4. 2 + \frac{\sqrt[3]{6} + \sqrt[3]{36}}{3}; \text{etc.}$$

$$5. -m + m\sqrt[3]{m} + \sqrt[3]{m^2}; \text{etc.}$$

Ex. 88. Page 479.

$$1. 3 \pm \sqrt{2}, 3 \pm \sqrt{2}.$$

$$4. 2 \pm \sqrt{-11}, \frac{7 \pm \sqrt{37}}{2}.$$

$$2. 2 \pm \sqrt{7}, -2 \pm \sqrt{2}.$$

$$5. 6 \pm \sqrt{23}, \frac{-5 \pm \sqrt{77}}{2}.$$

$$3. 2 \pm \sqrt{7}, 2 \pm \sqrt{-3}.$$

8. $I = 0$. Reducing cubic is $4a^3\theta^3 = G^3 + 4H^3$, and in equation (6), page 475, $\theta_1, \theta_2, \theta_3$, are the three cube roots of $\frac{G^3 + 4H^3}{4a^3}$.

$J = 0$. The reducing cube is $4a^3\theta^3 - Ia\theta = 0$, of which the roots are $0, \pm \frac{I}{2a}$. Therefore

$$ax + b = \sqrt{-H} + \sqrt{\frac{1}{2}aI - H} + \sqrt{-\frac{1}{2}aI - H}.$$

10. $\theta^3 - 3bc\theta - (b^3 + c^3) = 0$.

Ex. 89. Page 494.

1. $2a^4 - 12a^2b^2 + 2b^4$.

2. $\frac{4}{5}$.

3. 8.

6. $\frac{5}{34}$.

7. $\frac{1}{2}\sqrt[3]{2}(\sqrt{2+\sqrt{3}}+i\sqrt{2-\sqrt{3}});$
 $-\frac{1}{2}\sqrt[3]{2}(\sqrt{2-\sqrt{3}}+i\sqrt{2+\sqrt{3}});$
 $\frac{1}{\sqrt[3]{2}}(-1+i).$

8. Cf. Ex. 2, § 511.

9. $\pm(\sqrt{\frac{1}{2}\sqrt{5}+1}+i\sqrt{\frac{1}{2}\sqrt{5}-1});$
 $\pm(\sqrt{\frac{1}{2}\sqrt{5}-1}-i\sqrt{\frac{1}{2}\sqrt{5}+1}).$

10. 3.5844; -3.3300; -0.2544.

11. 1.212; -1.571; -1.1427.

